The charmed and bottom meson spectrum from lattice NRQCD

Randy Lewis[a] and R. M. Woloshyn[b]

[a]Department of Physics, University of Regina, Regina, SK, Canada S4S 0A2

[b]TRIUMF, 4004 Wesbrook Mall, Vancouver, BC, Canada V6T 2A3

The mass spectrum of S and P-wave mesons containing a single heavy quark has been computed using quenched lattice nonrelativistic QCD. Numerical results have been obtained at first, second and third order in the heavy quark expansion, so convergence can be discussed. The computed spectrum of charmed and bottom mesons is compared to existing model calculations and experimental data.

1. MOTIVATION

The masses of the ground state doublets of S-wave heavy-light mesons ($D^{(*)}$, $D_s^{(*)}$, $B^{(*)}$ and $B_s^{(*)}$) are well known experimentally, and thus provide a testing ground for the lattice method. In particular, they can be used to examine the convergence properties of the heavy quark expansion in nonrelativistic QCD (NRQCD),[1] which is the most economical lattice action available for heavy quark physics.[2]

The spectrum of P-wave heavy-light mesons is not yet understood, experimentally nor theoretically, so here the lattice can offer predictions. Do the masses follow the ordering that is observed in the hydrogen atom? What are the magnitudes of the mass splittings among P-waves — tens of MeV or hundreds?

Lattice studies of the P-wave spectrum have been performed for an infinitely-heavy quark[3]. A subset of the P-wave states have also been observed by using a relativistic (clover) action in the charmed region, and extrapolating to the bottom region[4]. Recently, the lattice NRQCD approach has been used to work directly with B^{**} mesons[5–7] as well as D^{**} mesons[5,7].

In the following sections, the lattice NRQCD method is briefly reviewed and the results of numerical simulations are presented. Implications for the convergence of the lattice NRQCD expansion are discussed, and predictions for the B^{**} and D^{**} mass spectra from various lattice simulations are compared to theoretical models.

2. LATTICE NRQCD METHOD

The application of lattice NRQCD to heavy-light mesons entails an expansion of the action in inverse powers of the heavy quark mass M,

$$S = S_{q,G} + \int d^4x\, Q^\dagger \left(iD_t + \frac{\Delta^{(2)}}{2M} - \delta H \right) Q, \quad (1)$$

where $S_{q,G}$ has no heavy quark fields, and[2,8]

$$\delta H = \delta H^{(1)} + \delta H^{(2)} + \delta H^{(3)} + O(1/M^4), \quad (2)$$

$$\delta H^{(1)} = -\frac{c_4}{u_s^4}\frac{g}{2M}\boldsymbol{\sigma}\cdot\tilde{\mathbf{B}} + c_5\frac{a_s^2\Delta^{(4)}}{24M}, \quad (3)$$

$$\begin{aligned}
\delta H^{(2)} = &\ \frac{c_2}{u_s^2 u_t^2}\frac{ig}{8M^2}(\tilde{\boldsymbol{\Delta}}\cdot\tilde{\mathbf{E}} - \tilde{\mathbf{E}}\cdot\tilde{\boldsymbol{\Delta}}) \\
&- \frac{c_3}{u_s^2 u_t^2}\frac{g}{8M^2}\boldsymbol{\sigma}\cdot(\tilde{\boldsymbol{\Delta}}\times\tilde{\mathbf{E}} - \tilde{\mathbf{E}}\times\tilde{\boldsymbol{\Delta}}) \\
&- c_6\frac{a_t(\Delta^{(2)})^2}{16nM^2},
\end{aligned} \quad (4)$$

$$\begin{aligned}
\delta H^{(3)} = &\ -c_1\frac{(\Delta^{(2)})^2}{8M^3} - \frac{c_7}{u_s^4}\frac{g}{8M^3}\left\{\tilde{\Delta}^{(2)}, \boldsymbol{\sigma}\cdot\tilde{\mathbf{B}}\right\} \\
&- \frac{c_9 i g^2}{8M^3}\boldsymbol{\sigma}\cdot\left(\frac{\tilde{\mathbf{E}}\times\tilde{\mathbf{E}}}{u_s^4 u_t^4} + \frac{\tilde{\mathbf{B}}\times\tilde{\mathbf{B}}}{u_s^8}\right) \\
&- \frac{c_{10}g^2}{8M^3}\left(\frac{\tilde{\mathbf{E}}^2}{u_s^4 u_t^4} + \frac{\tilde{\mathbf{B}}^2}{u_s^8}\right) \\
&- c_{11}\frac{a_t^2(\Delta^{(2)})^3}{192n^2M^3}.
\end{aligned} \quad (5)$$

The chromoelectric and chromomagnetic fields are $E_i = F_{4i}$ and $B_i = \epsilon_{ijk}F_{ijk}/2$ respectively, and Δ denotes the lattice derivative. The coefficients c_i are usually set to their classical values ($c_i = 1$) in lattice NRQCD simulations. This assumes that perturbative corrections at momentum scales beyond the lattice cutoff (i.e. the inverse lattice spacing) are small. Nonperturbative contributions to the c_i coefficients are not neglected, and appear explicitly in Eqs. (1-5) via the tadpole factors, mentioned below.

A tilde on any quantity indicates that the leading classical discretization errors have been removed. Besides this classical improvement, a mean-field form of quantum improvement is also implemented through the introduction of two "tadpole factors", u_s and u_t, following the well-known suggestion of Lepage and Mackenzie[9]. Notice that the action of Eqs. (1-5) allows different lattice spacings, a_s and a_t, in the spatial and temporal directions; hence two tadpole factors are required.

Various choices for the light quark and gauge field action, $S_{q,G}$ of Eq. (1), have been considered. In the lattice NRQCD simulations of Refs. [1,5], the gauge field portion is classically and tadpole-improved, whereas the authors of Refs. [6,7] chose the unimproved Wilson gauge field action. For the fermion portion, Refs. [1,6,7] employ the tadpole-improved clover action[10] while Ref. [5] uses a more highly improved D234 action[11]. When the heavy quark mass is sufficiently large and the lattice spacing sufficiently small (but not small compared to the inverse heavy quark mass), one expects the predictions of these actions to agree.

3. S-WAVE HEAVY-LIGHT MASSES

Lattice NRQCD simulations typically produce a clear signal for the S-wave heavy-light meson masses, so they are convenient observables to use for testing the heavy quark expansion. Since the leading contribution to the heavy quark mass does not appear explicitly in the NRQCD action, only mass differences (or more generally energy differences) can be computed. The absolute mass scale can be extracted from the kinetic energy of
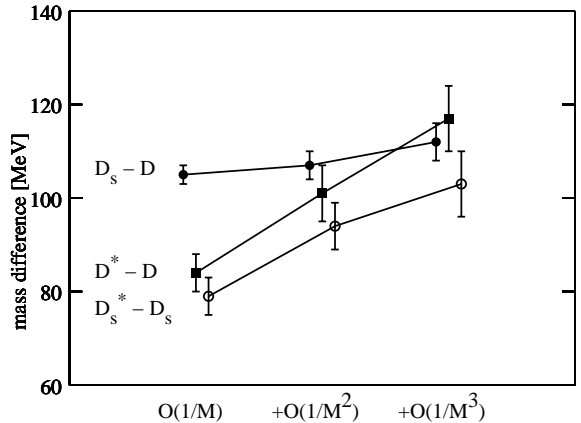

Figure 1. S-wave splittings of charmed mesons at first, second and third order in NRQCD from Ref. [5]. Solid lines simply connect related data points; they are not fits.

a meson,

$$E_{\mathbf{p}} - E_0 = \frac{\mathbf{p}^2}{2M_{\mathrm{kin}}} - \frac{\mathbf{p}^4}{8M_{\mathrm{kin}}^3} + \ldots, \qquad (6)$$

to the desired order in $1/M_{kin}$, where $E_{\mathbf{p}}$ and E_0 are taken from lattice simulations, and the "kinetic" mass M_{kin} is thus obtained.

The first three orders in the $1/M$ expansion of Eqs. (1-5) have been studied in Refs. [1,5] for two different light quark actions. The charmed results of Ref. [5] are displayed in Fig. 1. In this plot, subleading corrections to the $D_s - D$ splitting are insignificant, and the physical result agrees well with the experimental result[12],

$$\left[D_s - D^+\right]_{\mathrm{expt}} = 104 \text{ MeV}. \qquad (7)$$

The $D^* - D$ splitting of Fig. 1 has $O(1/M^2)$ and $O(1/M^3)$ corrections which are roughly equal, at about 20% each, and the $D_s^* - D_s$ splitting acquires 20% and 10% corrections at $O(1/M^2)$ and $O(1/M^3)$ respectively. Firm statements about convergence, or lack thereof, for the $1/M$ expansion are difficult to support with only these data. The first correction terms are not far from their expected size, $O(\Lambda_{\mathrm{QCD}}/M_{\mathrm{charm}})$, but the $O(1/M^3)$ corrections are perhaps larger than

Table 1
Quenched lattice NRQCD results for the S-wave bottom mesons[5] compared to experimental data[12], in units of MeV.

	quenched	experiment
$B_s - B$	92 ± 3	90 ± 2
$B^* - B$	25 ± 2	46
$B_s - B$	26 ± 1	47 ± 3

might have been hoped. These spin splittings are clearly smaller than the experimental values[12]

$$\left[D^{+*} - D^{+}\right]_{\text{expt}} = 141 \text{ MeV}, \qquad (8)$$

$$\left[D_s^* - D_s\right]_{\text{expt}} = 144 \text{ MeV}, \qquad (9)$$

as typical of quenched lattice calculations.[1,4–6]

The $1/M$ expansion is nicely suited to the bottom mesons and, even with 2000 gauge field configurations, the simulations of Ref. [5] found the nonleading contributions to be smaller than statistical uncertainties. The quenched results are shown beside the experimental data in Table 1.

4. P-WAVE HEAVY-LIGHT MASSES

It is more difficult to extract a signal for the P-wave masses from lattice simulations. Creation operators which extend over more than a single lattice site need to be tuned to enhance the signal.

In Ref. [5], the P-wave masses were studied at $O(1/M)$, $O(1/M^2)$ and $O(1/M^3)$ for both charmed and bottom mesons. Subleading contributions were found to be smaller than the statistical uncertainties. Fig. 2 compares the splitting between two P-wave masses (J=0 and J=2) as obtained from lattice calculations and from various models.[13] For the B^{**} system, most methods predict a splitting between zero and 80 MeV.

The discrepancy between the two lattice results is somewhat disconcerting. Both computations employed quenched lattice NRQCD and both had a temporal lattice spacing of about 0.1 fm. Ref. [5] used an improved gauge action with a spatial lattice spacing of 0.2 fm, a highly-improved (D234) light-quark action and an ensemble of 2000 configurations. Ref. [6] used the unimproved (Wilson) gauge action with a spatial
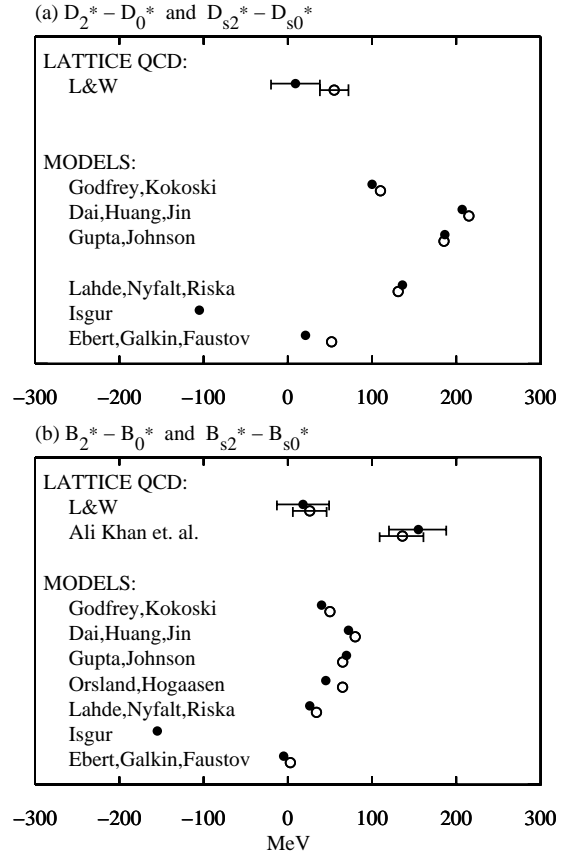

(a) $D_2^* - D_0^*$ and $D_{s2}^* - D_{s0}^*$

(b) $B_2^* - B_0^*$ and $B_{s2}^* - B_{s0}^*$

Figure 2. Splittings between the P_0 and P_2 masses for the D^{**} and B^{**} systems. Open(solid) symbols involve an s(u,d) quark.

lattice spacing near 0.1 fm, the improved clover action for light quarks and an ensemble of 102 configurations.

In Ref. [5], the ratio of P_2/P_0 correlation functions falls monotonically as Euclidean time increases, and of course statistical uncertainties grow as Euclidean time increases. Thus, if the meson mass is extracted from a region which begins too near the initial time, the predicted mass splitting would be too large. With this in mind, it is noted in Ref. [5] that, even at the smallest Euclidean times, those lattice data cannot reproduce the large splitting of Ref. [6]. Instead, Ref. [5] arrives at a plateau-independent upper bound of

study with lattice NRQCD. The results of Ref. [5] indicate that the D^{**} splittings are $O(50$ MeV$)$ or smaller, while the B^{**} splittings are $O(10$ MeV$)$ or smaller. However, the lattice work reported in Ref. [6] is not consistent with this, so further studies will be instructive.

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(a) $D_0^* - D$ and $D_{s0}^* - D_s$

LATTICE QCD:
L&W
Boyle
MODELS:
Godfrey,Kokoski
Dai,Huang,Jin
Gupta,Johnson
Lahde,Nyfalt,Riska
Isgur
Ebert,Galkin,Faustov

100   200   300   400   500   600   700   800

(b) $B_0^* - B$ and $B_{s0}^* - B_s$

LATTICE QCD:
L&W
Ali Khan et. al.
Boyle
MODELS:
Godfrey,Kokoski
Dai,Huang,Jin
Gupta,Johnson
Orsland,Hogaasen
Lahde,Nyfalt,Riska
Isgur
Ebert,Galkin,Faustov

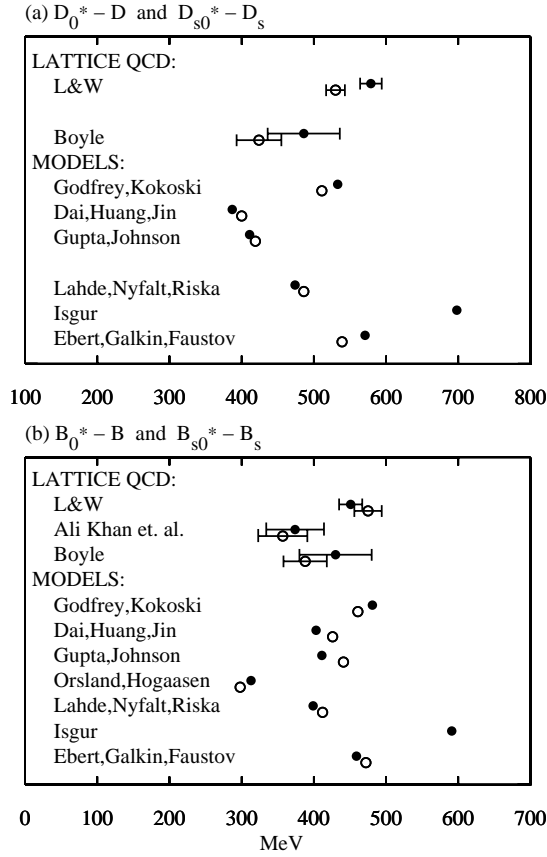0   100   200   300   400   500   600   700
MeV

Figure 3. S-P splittings for heavy-light mesons. Open(solid) symbols involve an s(u,d) quark.

100 MeV for the $B_2^* - B_0^*$ splitting. Lattice efforts are ongoing, and we look forward to a clearer understanding of the D^{**} and B^{**} spectra.

In Fig. 3, the splitting between S and P-wave mesons is shown. The lattice results of Boyle are interesting in that they do not rely on NRQCD.[4]

5. SUMMARY

The convergence of lattice NRQCD has been studied up to $O(1/M^3)$ for both S and P-wave heavy-light mesons. The only cause for concern seems to be the S-wave spin splitting for charmed mesons, where a definitive statement cannot yet be made about convergence.

The P-wave masses have recently come under